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RE-DESIGN OF THE TRANSMISSION SYSTEM OF THE EDISON ELECTRIC COMPANY OF SOUTHERN CALIFORNIA

by

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The system of the Edison Electric Co. in southern California as it exists today, is the result of the consolidation and growth of several smaller systems. A frequency of 50 cycles per second was adopted for the first plants, and so when new lines and new power stations were added it was almost a necessity to retain this frequency.

Plate I shows the general arrangement of the system as it is at present. The largest and longest transmission is from the Kern River Plant to Los Angeles. Here 20,000 K.W. is transmitted 116 miles. About 90 miles east of Los Angeles are four water-power plants, two on Mill Creek and two on the Santa Ana River. The aggregate capacity of these plants is 8,200 K.W. Most of this power however is used in the vicinity of Redlands and Colton, only a small part being transmitted to Los Angeles. At times even the direction of the flow of the power is reversed, the Kern River plant taking a part of the load east of Los Angeles. There is also a 500 K.W. water power plant on Lytle Creek. All of this power is used in its immediate vicinity however. At Los Angeles there is a large steam turbine station

of 10,000 K.W. capacity for use as reserve and in case of emergency. There are also two small steam plants, a 400 K.W. plant at Redlands and a 550 K.W. plant at Santa Monica. The lighting circuits are carried from the secondaries of step down transformers fed directly from the transmission lines and of course this means that there must be no violent fluctuations in voltage of the main transmission lines. Slow variation in voltage such as would be caused by change in the lighting load can be taken care of by hand regulation, but the violent fluctuations caused by heavy railway loads can be taken care of only at the expense of the power-factor of the system, i.e.Tirrill regulators could be installed on the A.C. ends of the motor-generator sets carrying the railway load or the exciters could be compounded with a part of the direct current supplied by the motor generators. Either of these methods means a lower average power-factor and so greater heating in the synchronous motors and transformers feeding them and lower line efficiency. If the railway load is only a small part of the total, as in this case, this point is not very important but there is a great deal of available railway load in the district around Los Angeles, and if the com-

pany was in a position to handle it to advantage they could probably get a considerable part of it. Hence the object of this investigation is to re-design the general features of the system on the assumption that all of the present electrical machinery is wiped out so that there will be no limitations of frequency or voltage imposed by existing apparatus. That is, it is proposed to determine the best frequency, voltages, sizes of high-tension lines, and general arrangement of stations. It is not intended to make a detailed design of any of the stations but simply to decide on the general arrangement of bus-bars, switches, transformers, etc. on account of the effect that it has on the comparative cost of the system at different voltages.

The following is an estimate of the loads at the different stations made by Mr. Ensign:

Station	Pumping	Factory	Lighting	Railway	Total
Redlands	700		400	200	900
Colton	1800			900	2200
Pomona	800		500		900
Fullerton	450		450		700
Pasadena		250	1300		1500
Los Angeles		2000	7000	9000	17000
Santa Monica, Inglewood, Redondo, Long Beach	500		5000		5000
Total	4250	2250	14650	10100	24000

Three phase transmission has so many advantages over any other number of phases that nothing else was considered and so all that follows applies to three phase transmission.

In the United States 25 and 60 cycles per second are the only standard frequencies so in deciding on the frequency of the system it is simply a question of deciding between 25 and 60 cycles. It is true that 15 cycles has been proposed for single phase railway work and has some advantages over 25 in this field, but up to date all, I think, of the single phase roads in operation have adopted 25 cycles; so even for single phase railways 25 cycles may be considered as standard.

For a system whose load is entirely lighting 60 cycles is undoubtedly the best. Both arcs and incandescents are very satisfactory at this frequency; transformers are smaller and cost less than at 25 cycles; and rather poor line regulation is not objectionable since the variations in load are slow enough to be followed accurately by hand adjustment of induction regulators or by the field rheostats of the generators.

On the other hand, if the current is used entirely for power purposes,25 cycles is much better. 25 cy-

cle induction motors have a much better power factor than 60 cycle motors, and this means that if a large number of induction motors are used the general power factor of the whole system will be much better at 25 cycles. Of course over-excited synchronous motors could be used to compensate for the lagging current, but 25 cycles would still have the advantage in that less over-excitation would be required. For getting 600 volt direct current for railway purposes, rotary convertors with transformers operating on 25 cycles are very satisfactory; and while 60 cycle rotaries are built for 600 volts, their design is necessarily inferior to 25 cycle rotaries, and motor generators are generally used instead. If single phase railway load is to be handled motor generator sets would be required with the 60 cycle transmission, while with 25 cycles the railway load could be carried directly from the transformer secondaries.

Hence it is seen that the choice of frequency is determined principally by the character of the load to be carried. The usual conditions, as in this case, are that a great variety of load is to be carried by the system, and so the best frequency is determined by the relative amounts of the different kinds of load and also

by the judgment of the engineer as to what kind of load may be expected in the future.

From the table previously given it is seen that about one half of the load is lighting and one half power; but on account of the rapid development of the electric railways around Los Angeles,it is probable that the power load will increase more rapidly than the lighting load. It is seen that the estimate of the total is not the sum of the separate maxima. This is on account of the peaks in the load curves of different kinds coming at different times. Power for irrigating, for instance, is sold at a very low rate on condition that the consumer is not to take current during the lighting peak.

If 60 cycles is used the best arrangement is probably the one in use at present, i.e. motor-generators for the railway work and Edison three wire D.C.; and the lighting and induction motor circuits fed through transformers from the main transmission lines.

If 25 cycles is used, rotary convertors with transformers could be used for the railway circuits and motor-generators converting to 60 cycles would be desirable for feeding the lighting circuits. Of course the incandescant circuits could be operated at 25 cycles but if this was done the regulation of the main trans-

mission lines would have to be very good, since incandescent lamps flicker perceptibly on 25 cycles when the voltage is a little low, and it would be more satisfactory to install motor generators for converting to 60 cycles and not attempt to regulate the 25 cycle system accurately. This puts the regulation of the lighting circuits under the control of the substation feeding them and so is independent of the regulation of the main transmission lines. However the voltage of the D.C. end of the rotaries feeding the railway circuits is not capable of independent control in this way, but since the regulation of the railway circuits is not nearly so important as the lighting circuits this point is not objectionable. With the lighting circuits fed directly from the main transmission lines and motor-generators used for the railway circuits, the railway circuits are capable of being regulated accurately but the lighting circuits are not independent of the fluctuations caused by the railway load, except as before stated at the expense of the power factor. With motor-generators feeding the lighting circuits, however, the regulation is where it belongs, and the company is in a position to give good lighting service and still handle

large railway loads economically. Plate II gives the regulation of the Kern River line at both 25 and 60 cycles at 60000 volts receiver pressure. Plate III gives the same at 80,000 volts. The diagrams are all drawn to the same scale so the relative size gives a clear idea of the effect of changing frequency. Obviously in so far as regulation is concerned 25 cycles is far the better.

The following is a summary of the relative advantages of the two systems, 25 cycles using rotaries for the railway load and 25-60 cycle motor generators for the lighting load; and 60 cycles using motor generators for the railway load and feeding the lighting circuits, through transformers, from the main transmission lines.

25 Cycles.	60 Cycles.
Better regulation of lighting circuits when there is a large railway load.	Better regulation of railway circuits.
Lower first cost for railway load.	Lower first cost for lighting load.
Better regulation of transmission line. Smaller	

25 Cycles.	60 Cycles.
charging current.	
Induction motors have better power factor.	
Mixed load can be handled better.	

25 cycles was selected as the best frequency for this case. It was also decided to use rotary convertors for getting 600 volt direct current for railway purposes, and synchronous motor-generator sets for carrying the lighting load. 62½ cycles was selected instead of 60 for the lighting circuits because of the greater variety of possible motor-generator speeds. With 25 to 60 cycle motor generator sets the only possible speeds are the sub-multiples of 300 while, if 62½ cycles is used the available speeds are the sub-multiples of 750. Standard 60 cycle apparatus can be used on the 62½ cycle circuits since, at the slightly higher frequency the core loss and magnetizing current of transformers, etc. will be less, and the increase in frequency is not enough to interfere with such small motors as might be operated from the lighting circuits.

Before considering the question of voltage it

will be necessary to decide upon the scheme of transformer connections to be used, since the cost of transformers depends upon their voltage, which in turn depends upon whether star or delta connection of the high-tension windings is to be used. The scheme decided upon was star connection on the high-tension side and delta on the low, with the high-tension neutral grounded. It has many advantages over other combinations of star and delta connections. One transformer or one line can be cut out, and the two remaining will deliver three-phase power to the low-tension circuit. If three-phase rotaries, or six-phase rotaries using double delta connected transformer secondaries are operated from the low-tension side, they will operate just as well with one line wire out except that the regulation will not be quite so good. If six phase rotaries with diametrically connected transformer secondaries are used, cutting out one transformer will cut off the power supply from one diameter, and so increased heating will result, but full load can still be carried.

Transformers connected delta or both high and low tension sides give the same flexibility as regards cutting out single transformers, but the three lines

must be in all of the time to deliver three-phase power. However delta connected transformers have the advantage in that cutting out a transformer does not cut out a high tension wire and so does not affect the line regulation. This advantage is slight compared with the ability to operate with one high tension wire cut out, since cases of line trouble are of far more frequent occurrence than cases of transformer trouble.

With the star high tension and delta low tension connection there is no chance of getting abnormally high voltages by direct transformation as long as the ground connections are maintained and there is no possibility of getting excessive voltages by resonance unless a transformer becomes disconnected from the low tension delta with the high tension side still in. Hence the low tension transformer leads should not be fused. When one line is cut out the earth forms the third leg of an inverted three-phase system. In this case each of the remaining line wires carries $\sqrt{3}$ times as much, and the earth the same current as was carried originally by each line wire. Hence if we neglect the drop in the earth the line drop will be 73% greater than before.

The selection of the best voltage for a given transmission depends upon the allowable drop and economic conditions subject to engineering limitations. Since very accurate regulation is not important in this case on account of using motor-generator sets for lighting, it was found that if the economic conditions were satisfied, the drop was well within limits set by operating conditions. That is, the best voltage to use is the one which will make the cheapest system both to construct and to operate, due account being taken of all factors which enter into the costs, such as repairs, depreciation and reliability. A certain case might apparently require a voltage considerably higher than what is considered standard practice, but increased cost of repairs and increased trouble due to the excessive voltage might easily make the lower voltage more desirable.

The standard average high-tension voltages recommended by the last report of the Standardization Committee of the American Institute of Electrical Engineers are as follows.

6600 - 11000 - 22000 - 33000 - 44000 - 66000 - 88000

So any other than these was not considered.

The economical current density was determined as follows. The method is really a statement of Kelvin's Law.

Let i = Square root of mean square current, i.e. the current which is flowing continually for the year would give the same copper loss as the actual current.

r = Resistance of the conductor considered.

l = Length in feet of the conductor considered.

c.m. = Circular Mills cross section of the conductor.

c_1 = Cost of electrical energy per K.W.Hr.

c_2 = Cost of copper per lb.

a = Rate per yr. which the company earns on its actual investment, unless this is less than the rate for which money can be borrowed.

Then for copper conductors $r = \frac{10.35\ l}{c.m.}$

K.W.Hrs. lost per year $= \frac{i^2 r \times 24 \times 365}{1000} = \frac{10.35 \times 24 \times 365}{1000} \times \frac{l i^2}{c.m.}$

Cost of energy lost per yr. $= \frac{10.35 \times 24 \times 365}{1000} \times \frac{l i^2 c_1}{c.m.}$

Cost of copper $= 3.02 \times 10^{-6} c_2\ l\ c.m.$

Total cost per year = T $= \frac{10.35 \times 24 \times 365}{1000} \times \frac{l i^2 c_1}{c.m.} + 3.02 \times 10^{-6} a c_2\ l\ c.m.$

Let $K_1 = \frac{10.35 \times 24 \times 365}{1000}$

$$K_2 = 3.02 \times 10^{-6}$$

$$\text{Then } T = K_1 \frac{li^2 c_1}{c.m.} + K_2 a c_2 l c.m.$$

F or T minimum

$$\frac{dT}{dc.m.} = 0$$

$$\text{Hance } \frac{dT}{dc.m.} = -\frac{K_1 l i^2}{(cm)^2} + K_2 a c_2 l = 0$$

$$\frac{cm}{i} = \sqrt{\frac{K_1}{K_2}\,\frac{c_1}{ac_2}} = .5479 \sqrt{\frac{c_1}{ac_2}}$$

In the above a is taken as the percentage earned by the company rather than the rate for which money can be borrowed because a company would not care to invest money in additional copper unless it would bring at least as high a rate of interest in cost of power saved as was being earned by the rest of their capital. Otherwise increasing the investment would decrease the percentage earnings.

This equation gives the economical current density in circular mills per square root of mean square ampere. To reduce this to circular mills per max. ampere it must be multiplied by ratio of the square root of the mean square to the maximum. This ratio will be somewhat larger than the load factor, which is the ratio of the average to the maximum, since in general the square root

of the mean square is greater than the average. The ratio of the square root of the mean square to the average was determined for two of the curves shown in Plate IV. This was 1.07 for one and 1.075 for the other. Hence since the load factor for the month is 50.9% the ratio of the square root of the mean square to the maximum for the month will be about 7% greater or 54.5%. For the year this would be somewhat less, but on the contrary the load factor of the main transmission lines from the water-power plants and hence the ratios of the square root of the mean square to the maximum, will be greater because of the steam power used to help out at the peak load. Also the current density ought to be somewhat less than given by the above considerations since there will be times when one transmission line is cut out for repairs and the remaining lines will carry all of the load. This would mean more energy loss than is accounted for by the equations and hence a larger economical cross section of copper. So if we take the ratio of the square root of the mean square to the maximum as 60% we will about take care of the conditions.

Taking $c_1 = 0.003$ $\quad \frac{cm}{I} = 2632$
$c_2 = 0.13$
$a = 0.10$

And $\frac{cm}{I_{max}} = 0.60 \times 2632 = 1579$

For the lines between Redlands and Los Angeles where the direction of the flow of power reverses the ratio of the square root of the mean square to the maximum is very uncertain; but 60% was taken on this as well as on all of the other lines. This makes the economical current density,as derived above, 1579 circular mills per ampere.

A preliminary investigation of the Kern River line showed that 44,000 volts or anything less would not do because of the very rapid increase in the cost of copper. At this voltage also the drop becomes the limiting condition so that the cross section of copper would have to be increased beyond the economical density adding still further to the disadvantage of the lower voltage. Hence for the Kern River line the selection of the economical voltage becomes a question of choosing between 66,000 and 88,000 volts average which is about 60,000 or 80,000 at the receiving end of the line. The following is an estimate of the comparative cost of the line at 66,000 and 88,000 volts. Only those items are included which are affected by a change in voltage. The difference in weight of the copper would make practically no difference in the cost of steel towers so this is not

included. Also oil switches would cost nearly the same at either voltage so they are left out. For the cost of transformers the following formula was used:

$$c = 1.30 \left(\frac{E}{1000} + 55 \right) \sqrt{K.W.} \ \#$$

88,000 volts.

(116 + 2%) x 6 miles	#0 Copper @ (13 + 1½)¢	173624
13 - 1670 K.W. Transformers	(50810 volts)	73069
13 - 1550 K.W. "	(46190 volts)	67334
7200 Insulators	$5.70	41040
		$355067

66,000 volts.

(116 + 2%) x 6 miles	#00 Copper @ (13 + 1½)¢	218890
13 - 1670 K.W. Transformers	(38100 volts)	64298
13 - 1550 K.W. "	(34680 volts)	59616
7200 Insulators	$3.00	21600
		$364404

At 88,000 volts the cost of the power wasted per year under the conditions assumed would be $15688 while at 66,000 volts it would be$20325.The difference capitalized at 10% is $49370 which means that the 88,000 volt line could cost $49370 more than the 66,000 volt line and still be equally economical. Hence the total advantage of the 88,000 volt line over the 66,000 volt

Mershon in paper given at A.I.E.E. meeting, Dec.23, 04.

one is $58707 according to computations. However, the question of reliability and the cost of additional repairs due to the higher voltage is not taken into account and is very hard to estimate. Also the loss of power through brush discharge and leakage over insulators, etc. is not taken into consideration. All of these will decrease the apparent advantage of 88,000 volts and as 88,000 volts is considerably beyond present standard practice it was decided to use 66,000.

For the other lines it was seen that the choice was to be between 33,000 and 44,000 volts. The maximum power transmitted over the lines between Redlands and Los Angeles was estimated by Mr. Ensign to be 4,000 K.W. At 44,000 volts two circuits of #4 gives about the right current density, and since #4 is the smallest size which it is advisable to put up for mechanical reasons, 44,000 volts is obviously the highest voltage to be considered even if it were not for the shorter lines. So this part of the system was designed for 33,000 and 44,000 volts.

The following is an estimate of the cost at 33,000 volts of the items depending upon the voltage. The resistance drop in percent is also given for 100% power factor.[1]

1. Formula used $P = \frac{54.7 \cdot D \cdot KW}{E^2 \cdot CM \cdot \cos\phi} \cdot 10^8$ where D Distance in miles.

33,000 volts delta 19050 star.

Line	K.W.	Amperes	Dist in M.	% drop at 100% P.F.	Copper		Cost
Redlands to Colton	6600	115	9.0	1.78	2 circuits	#1	$10681
Colton to Los Angeles	4000	70	57.1	10.9	2 "	#3	42619
Puente to Los Angeles	2750	48.1	69.5	5.74	1 "	#1	41242
S.A.R.#2 to Redlands	4200 each way	73.5	12.0	2.40	2 "	#3	8957
S.A.R.#1 to S.A.R.#2	3000	52.5	2	0.36	2 "	#4	1184
M.C. #1 to Redlands	4000	70.0	9.5	1.82	2 "	#3	7090
M.C.#2 and#3 to M.C.#1	3250	56.8	2.0	0.39	2 "	#4	1184
Los Angeles to Pasadena	1500	26.2	10.	0.90	2 "	#4	5922
							$118879

Transformers.

10-83 K.W.	Mill Creek #1	$ 8764
13 - 250 K.W. 3 - 83 K.W.	Mill Creek #2 and #3	19773 2629
13 - 250 K.W.	S.A.R. #1	19773
17 - 200 K.W.	S.A.R. #2	23128
3 - 67 K.W.	Redlands	2362
3 - 233 K.W.	Redlands	4405
6 - 167 K.W.	Colton	7459
3 - 600 K.W.	Colton	7069
3 - 300 K.W.	Pomona	4999
3 - 500 K.W.	Pasadena	6453
3 - 250 K.W.	Fullerton	4563
20 - 250 K.W.	Santa Monica, Long Beach, etc.	30421
		$141798

Total High Tension Copper	$118879
Total Transformer	141798
9000Line Insulator 66¢	5940
	$266617

At 44,000 volts the cost of the same item will be:

44,000 volts delta 25,400 star.

Line	K.W.	Amperes	Dist. in M.	% drop at 100% P.F.		Copper	Cost
Redlands to Colton	6600	86.6	9.0	1.26	2	circuits #2	$ 8472
Colton to Los Angeles	4000	52.5	57.1	7.73	2	" #4	33817
Puente to Los Angeles	2750	36.1	69.5	5.12	1	" #3	25937
S.A.R.#2 to Redlands	4200 each way	55.1	12.0	1.70	2	" #4	7107
S.A.R.#1 to S.A.R.#2	3000	39.4	2.0	0.20	2	" #4	1184
M.C.#1 to Redlands	4000	52.5	9.5	1.28	2	" #4	5627
M.C.#2 and#3 to M.C.#1	3250	42.6	2.0	0.22	2	" #4	1182
Los Angeles to Pasadena	1500	19.7	10.0	0.51	2	" #4	5922
Transformers $\frac{80.4}{74}$ x Cost at 33,000 volts							153953
9000 Insulators 1.21							10890
						Total	$254093

Hence it is seen that the 44,000 volt system will cost $12524 less to construct than the 33,000 volt system and since the current density is the same, the 44,000 volt system will also be more economical to operate. Hence 44,000 volts was decided upon as the voltage to use in this case. It will be noticed that no transformers are included in this estimate for the Los Angeles station. It was planned to take out taps from the 60,000 volt transformers at the proper points to get 44,000 volts. This removes any apparent disadvantage in using the two high-tension voltages on different parts of the system. Transformers of sufficient capacity to handle all of the Kern River power would have to be installed at Los Angeles anyway so that modifying them slightly to get 44,000 volts would only involve a small additional expense and in so far as the Los Angeles transformers are concerned it makes practically no difference whether the Kern River voltage is carried to the system east of Los Angeles or not. Of course the transformers would have to be designed with this in view. The coils near the 44,000 volt taps would have to be re-enforced to stand the strains due to surges coming in over the 44,000 volt lines, and the coils in the two sections of the

high-tension winding would have to be interspersed so as to avoid excessive magnetic leakage when the transformer is acting as an auto-transformer converting from 60,000 to 44,000 volts.

In Plates II and III are shown the regulation diagrams of the Kern River Line at both 25 and 60 cycles and 60,000 and 80,000 volts. The method is Baum's.# a b represents the E.M.F. necessary to overcome the resistance of the line with full load current at 100% power factor. This is in phase with the current. bc represents the E.M.F. necessary to force the same current through the inductance of the line. This leads the current by 90° since it is in opposition to the E.M.F. of self-induction. Hence, ac, which is the vector sum of ab and bc, represents the total E.M.F. both in phase and amount necessary to force full load current at 100% power-factor through the line. Of course this all applies to one leg of the three phase circuit only. If there is also a wattless component of the current, the E.M.F. necessary to force this through the impedance of the line will lead or lag ac by 90° depending upon whether the current is leading or lagging respectively. Since, the ratio of the wattless component to the energy com-

Elec. Wld.& Engr. May 18, 1901.

ponent of the current is the tangent of the angle whose cosine is the power-factor, we can lay off the drop due to the wattless component of the load cd or cd' by making the angle cad or cad' = $\cos^{-1}$ P.F. (cad if the current is leading and cad' if the current is lagging). Then ad represents the total E.M.F. necessary to force the current over the line. Superimposed on this, however, is the charging current and if we assume that a single condenser at the middle of the line represents the effect of the disturbed capacity sufficiently accurately, and also assume that the charging current is in quadrature with the receiver pressure, the triangle aek represents the drop due charging current. It is similar to the large triangle cba but shifted 90^{o} ahead since the charging current is leading. ae represents the inductive drop and ek the resistance drop, hence aek represents the total drop due to the charging current. Then the pressure necessary to be maintained at the generator end of the line will be the resultant of receiver pressure, which in phase with ab, the line drop due to current delivered to the receiving circuit ad, and the line drop due to the charging current ak. This is obtained graphically by laying off the receiver vol-

tage from k to the left on the horizontal line to o (not shown). Then the distance od represents the generator voltage necessary to be maintained. This voltage is read from the diagram by drawing arcs of circles across the diagram all with the center at o and radii corresponding to different generator voltages. If the load is anything other than full load the line ac is shortened to ac' such that the ratio of the load considered to full load is the ratio of ac' to ac. The angles cad, etc. are laid off so that the diagram gives very easily the regulation of the line for any load and any power-factor.

The following are the calculations for determining the constants used in constructing the diagrams:

Kern River Line. 60,000 volts receiver pressure. Two circuits #00 spaced 6' c. to c. -116 miles+2% =118.3 mi.

Resistance = 0.410 x 118.3 = 48.50 Ohms.

Reactance[1] at 60 cycles $= \frac{0.280 \times 5.28 \times 118.3}{13} = 101.0$ Ohms.

Reactance at 25 cycles $= \frac{25}{60} \times 101.0 = 42.08$ Ohms.

Capacity $= \frac{0.0776 \times 118.3}{2 \log_{10} \frac{72}{0.209}} = 1.81$ Microfarads.

L. Fisher Trans. A.I.E.E., June 1905.

60 Cycles.

E_1 = 60,000 delta = 34,641 star.

I_1 = 86

I_c = Charging current

$$= \frac{63000 \times 2 \times 60 \times 1.81}{\sqrt{3} \times 10^6}$$

= 24.8 amp.

I_1R = 4171

I_1X = 8686

$\frac{I_cR}{2}$ = 601

$\frac{I_cX}{2}$ = 1252

At 100% P.F.

E_o = 34641-1252+4171 -

-(601+8686)j = 38690

K W_1 = 8938

$\frac{3.\ I_1^2R}{1000}$ = 1077

$\frac{I_c^2R}{1000}$ = 30

K_1W_o 10045

25 Cycles.

E_1 = 60000 delta = 34641 star.

I_1 = 86

$I_c = \frac{63000 \times 2 \times 25\ 1.81}{\sqrt{3} \times 10^6}$

= 10.3 amp.

I_1R = 4171

I_1X = 3619

$\frac{I_cR}{2}$ = 250

$\frac{I_cX}{2}$ = 217

At 100% P.F.

E_o = 34641-217+4171

-(250+3619)j = 37888

K W_1 = 8938

$\frac{3\ I_1^2R}{1000}$ = 1077

$\frac{I_c^2R}{1000}$ = 5

K.W_o 10020

Kern River Line at 80,000 volts receiver pressure.

Two circuits #0 - 8'c to c. -116 miles+2% sag. = 118.3 mi.

Resistance = 0.518 x 118.3 = 61.28 Ohms.

Reactance at 60 cycles $\frac{0.298 \times 5.28 \times 118.3}{\sqrt{3}}$ = 107.5 Ohms.

Reactance at 25 cycles $= \frac{25}{60} \times 107.5 = 44.78$ Ohms.

Capacity $= \frac{0.0776 \times 118.3}{2 \log_{10} \frac{96}{0.186}} = 1.69$ Microfarads.

60 Cycles.	25 Cycles.
$E_1 = 80000$ delta $= 46188$ star.	$E_1 = 80000$ delta $= 46188$ star.
$I_1 = 66$	$I_1 = 66$
$I_c = \frac{83000 \times 2 \times 60 \times 1.69}{\sqrt{3} \times 10^6}$	$I_c = \frac{83000 \times 2 \times 25 \times 1.69}{\sqrt{3} \times 10^6}$
$= 30.53$	$= 12.72$
$I_1R = 4044$	$I_1R = 4044$
$I_1X = 7095$	$I_1X = 2955$
$\frac{I_cR}{2} = 935$	$\frac{I_cR}{2} = 390$
$\frac{I_cX}{2} = 1641$	$\frac{I_cX}{2} = 285$
$K.W_1 = 9145$	$K.W_1 = 9145$
$\frac{3 I_1^2R}{1000} = 800$	$\frac{3 I_1^2R}{1000} = 800$
$\frac{I_c^2R}{1000} = 57$	$\frac{I_c^2R}{1000} = 10$
$K.W_o = 10002$	$K.W^o = 9955$

The following is the exact solution of the general transmission line problem taking account of distributed capacity.#

Merriman & Woodward, Higher Mathematics, p. 153.

Let E = E.M.F. at any point x in the line, x being measured from the receiving end.

I = Current at any point x.

R = Resistance of line in ohms per mile.

L = Coefficient of self-inductance in henrys per mi.

G = Coefficient of leakage in mhos per mile.

C = Capacity in farads per mile.

W = 2 π times frequency.

Then $dI = E(G-jWC)dx$

$$\frac{dI}{dx} = E(G-jWC) \qquad (1)$$

also $dE = I(R-jWL)dx$

$$\frac{dE}{dx} = I(R-jWL) \qquad (2)$$

Differentiating (1) abd (2) we have

$$\frac{d^2I}{dx^2} = \frac{dE}{dx}(G-jWC) \qquad (3)$$

$$\frac{d^2E}{dx^2} = \frac{dI}{dx}(R-jWL) \qquad (4)$$

Substituting (2) in (3)

$$\frac{d^2I}{dx^2} = I(R-jWL)(G-jWC)$$

And substituting (1) in (4) we have

$$\frac{d^2E}{dx^2} = E(R-jWL)(G-jWC)$$

Putting $m^2 = (R-jWL)(G-jWC)$ we have

$$\frac{d^2E}{dx^2} = m^2E \text{ --- ---------- ------ ----------} \quad (5)$$

$$\frac{d^2I}{dx^2} = m^2I \text{ ----------} \quad (6)$$

Hence E and I are functions differing in their limiting conditions only.

From (5)

$$\frac{d^2E}{dx^2} - m^2E = 0$$

$$\left(\frac{d}{dx} + m\right)\left(\frac{d}{dx} - m\right)E = 0$$

$$E = A \cosh mx + B \sinh mx \text{ ----------} \quad (7)$$

Similarly from (6)

$$I = A' \cosh mx + B' \sinh mx \text{ ----------} \quad (8)$$

Substituting the value of E from (7) in (1) we have

$$\frac{dI}{dx} = (A \cosh mx + B \sinh mx)(G-jWC)$$

Differentiating (8) and equating the coefficients of cosh mx and sinh mx in the two values of $\frac{dI}{dx}$ we have

$$A(G-jWC) = mB' \text{ and } B(G-jWC) = mA'$$

Hence $B' = \frac{A(G-jWC)}{m}$ and $A' = \frac{B(G-jWC)}{m}$

Or if we let $m_1 = \frac{m}{G-jWC}$

$$B' = \frac{A}{m_1} \qquad A' = \frac{B}{m_1}$$

Hence there are only two arbitrary constants. If E_1 and I_1 are the E.M.F. and current respectively at the receiving end of the line, substituting $E = E_1$, $I = I_1$, and $x = o$ in eqs. (7) and (8) we have

$$A = E_1 \quad \text{and} \quad A' = I_1$$

Hence $E = E_1 \cosh mx + m_1 I_1 \sinh mx$ ----------(9)

$$I = I_1 \cosh mx + \frac{E_1}{m_1} \sinh mx \text{----------(10)}$$

For one circuit of the Kern River line at full non-inductive load at 60,000 volts receiver pressure and 60 cycles with #00 copper we have the solution as follows:

The constants are $R = 0.410$

$$wL = 0.8536$$

$$G = 0 \text{ (Assumed)}$$

$$wC = 5.768 \times 10^{-6}$$

$$E_1 = 34641$$

$$I_1 = 86$$

$$m^2 = (R - jWL)(G - jWC)$$

$$= (0.410 - 0.8536j)(-5.768 \times 10^{-6}j)$$

$$= -4.923 \times 10^{-6} - 2.365 \times 10^{-6}j$$

This represents a vector of amplitude 5.458×10^{-6} and phase angle $205^0\text{-}39'$. Hence m will be a vector of amplitude $\sqrt{5.458 \times 10^{-6}}$ and phase angle $\frac{205^0\text{-}39'}{2}$. Resolv-

ing this into real and imaginary components

$$m = -5.183 \times 10^{-4}\ 2.278 \times 10^{-3}j$$

Also $m_1 = \frac{m}{G-jWC} = \frac{-5.183 \times 10^{-4}\ 2.278 \times 10^{-3}j}{-5.183 \times 10^{-6}j} = -395.0-89.85j$

Substituting in eq. (8) and putting $x = 118.3$ we have for the generator voltage

$$E_o = 34641 \cosh(-5.183 \times 118.3 \times 10^{-4} + 2.278 \times 118.3 \times 10^{-3}j) + 86x(-395.0-89.85j) \sinh(-5.183 \times 118.3 \times 10^{-4} + 2.278 \times 118.3 \times 10^{-3}j)$$

For getting the hyperbolic sines and cosines of complex numbers we have the following formula:#

$$\cosh(x+jy) = a+jb$$

$$\sinh(x+jy) = c+jd$$

where $a = \cosh x \cos y$

$b = \sinh x \sin y$

$c = \sinh x \cos y$

$d = \cosh x \sin y$

In this case $x = -5.183 \times 10^{-4} = -0.06131$

$y = 2.278 \times 118.3 \times 10^{-3} = 0.2695 = 15^{\circ}26'$

Hence $a = \cosh(-0.06131)\cos 15^{\circ}26' = 0.9658$

$b = \sinh(-0.06131)\sin 15^{\circ}26' = +0.01633$

$c = \sinh(-.0.6131)\cos 15^{\circ}26' = -0.05914$

$d = \cosh(-0.06131)\sin 15^{\circ}26' = 0.2666$

Merriman & Woodward, Higher Mathematics, p. 143.

Then $E_0 = 34641(0.9658-0.01633j)+86(-395.0-89.85j)(-0.05914+0.2666j) = 33455-556j+2009-9056j+457j+2060$

$= 37524-9165j = 38628$

The approximate method gives 38690 volts so it will be seen that the approximate method is plenty close enough for ordinary purposes. 60 cycles was purposely taken because the discrepency between the two methods would be much greater than at 25 cycles. Both of the methods, of course, are based on the assumption that both current and pressure waves are sinusoidal at all points. The error introduced by making this assumption would probably be far greater than that caused by the use of the approximate solution.

For zero current the above expressed becomes $E_0 = 33455-566j$. Hence in the diagram the line ae should be 1186 instead of 1252 as given by the approximate method. Also ek should be 566 instead of 601. Examining the products arising from the second term it is seen that to make the diagram correct ab should be 4069 instead of 4171 and bc should be 9599 instead of 8686.

The "unit" system is used in all stations as far as practicable. That is, in the power stations for instance each generator has its own bank of transformers

which are cut out when the generator is so the transformer capacity in operation is always equal to the generator capacity. No low tension bus-bar is provided, but duplicate high-tension bus-bars are installed. This will enable a single generator to be connected to a high tension line alone through its transformers so that the voltage can be brought up gradually if desired for testing out in case of line trouble. This also gives the operator a chance to work on the oil switches without shutting down anything. Of course it would be desirable to install duplicate exciter bus-bars as well, so that when testing defective lines the generator used could be separately excited and its voltage controlled by means of the exciter field rheostat since ordinarily the main field rheostats are not large enough to lower the voltage the desired amount.

Plates V to XV show the arrangement of the main 25 cycle wiring in the several stations. No attempt has been made to design the 62½ cycle system at all, and simply the bus-bar arrangement of the 11,000 volt 25 cycle system has been shown. 25 cycle generators are used in all of the stations excepting the following 62½ cycle machines:

6000 K.W. at Los Angeles.

400 K.W. at Redlands.

550 K.W. at Santa Monica.

Plate XVI shows the sizes and voltages of the main transmission lines as re-designed.

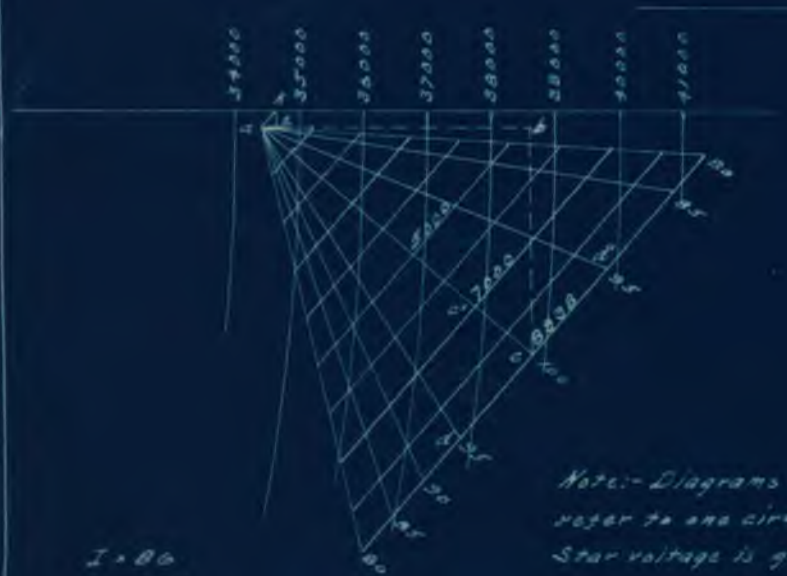
Approved by O. H. Ensign

June 10th 1908

REGULATION DIAGRAMS
KERN RIVER LINE
2 CIRCUITS #00 60000 VOLTS

25 CYCLES.

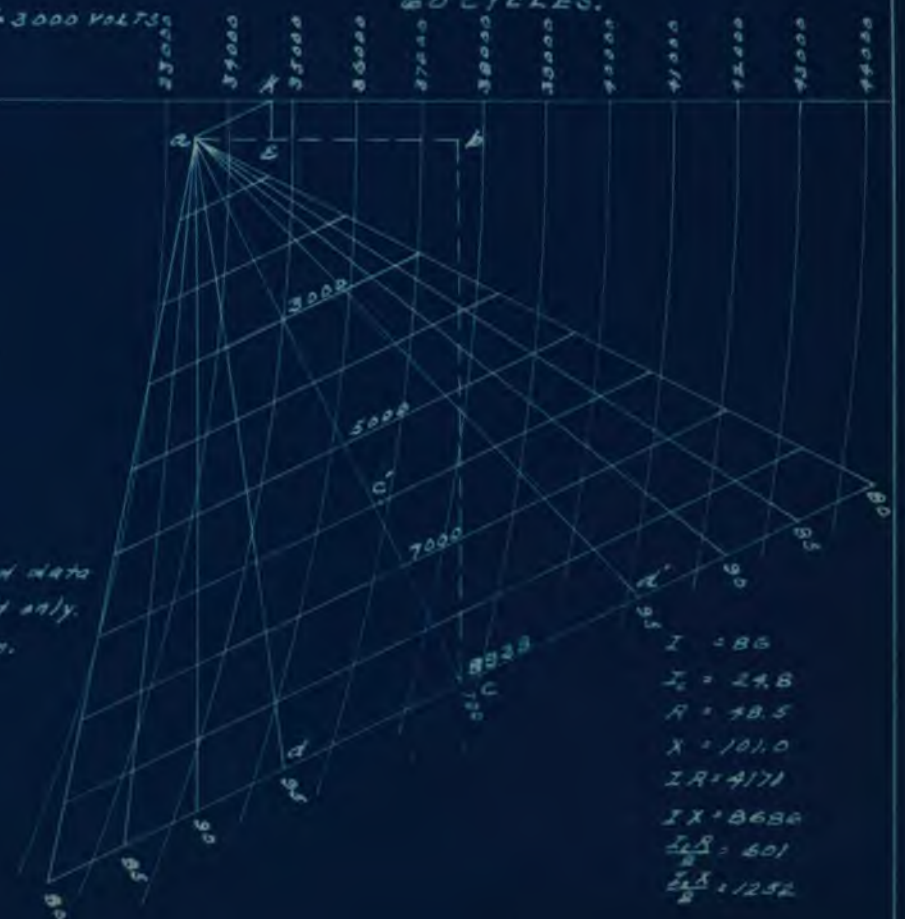
SCALE - 1" = 3000 VOLTS



Note:- Diagrams and data refer to one circuit only. Star voltage is given.

I = 80
I_c = 10.3
R = 48.5
X = 142.08
IR = 4171
IX = 3610
$\frac{E_s}{E_r}$ = 2.50
$\frac{E_s}{E_r}$ = 2.17

60 CYCLES.



I = 80
I_c = 24.8
R = 48.5
X = 101.0
IR = 4171
IX = 8686
$\frac{E_s}{E_r}$ = 601
$\frac{E_s}{E_r}$ = 1252

PLATE II

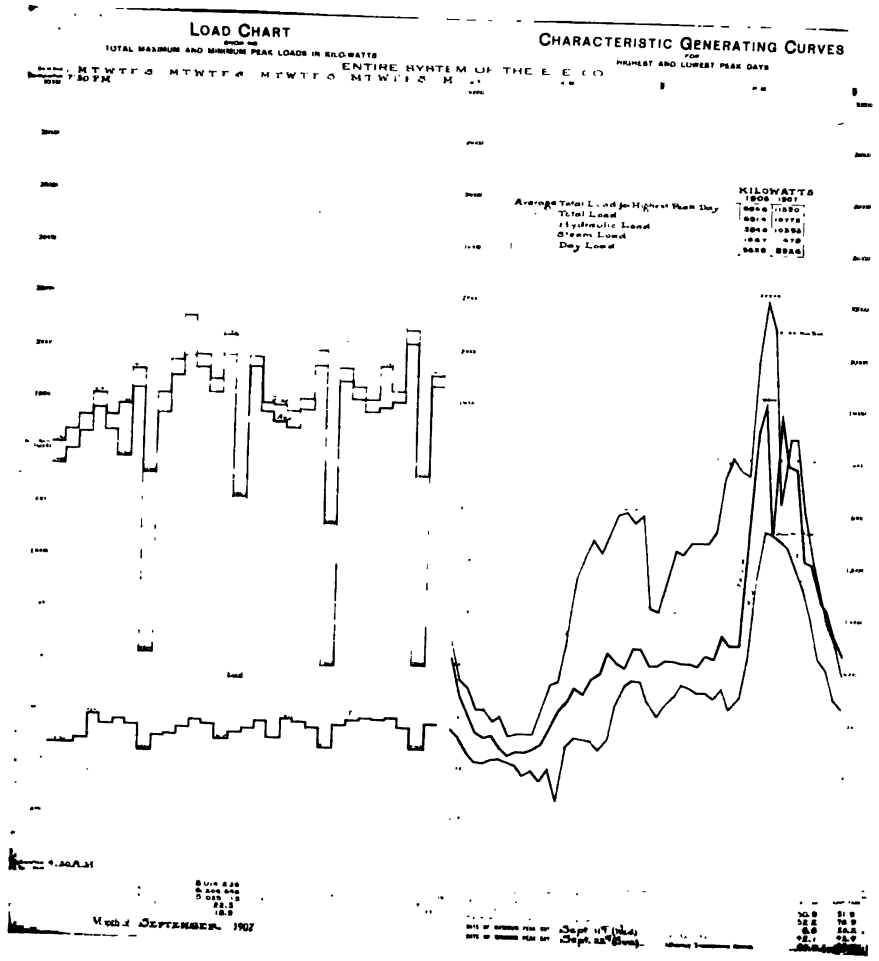
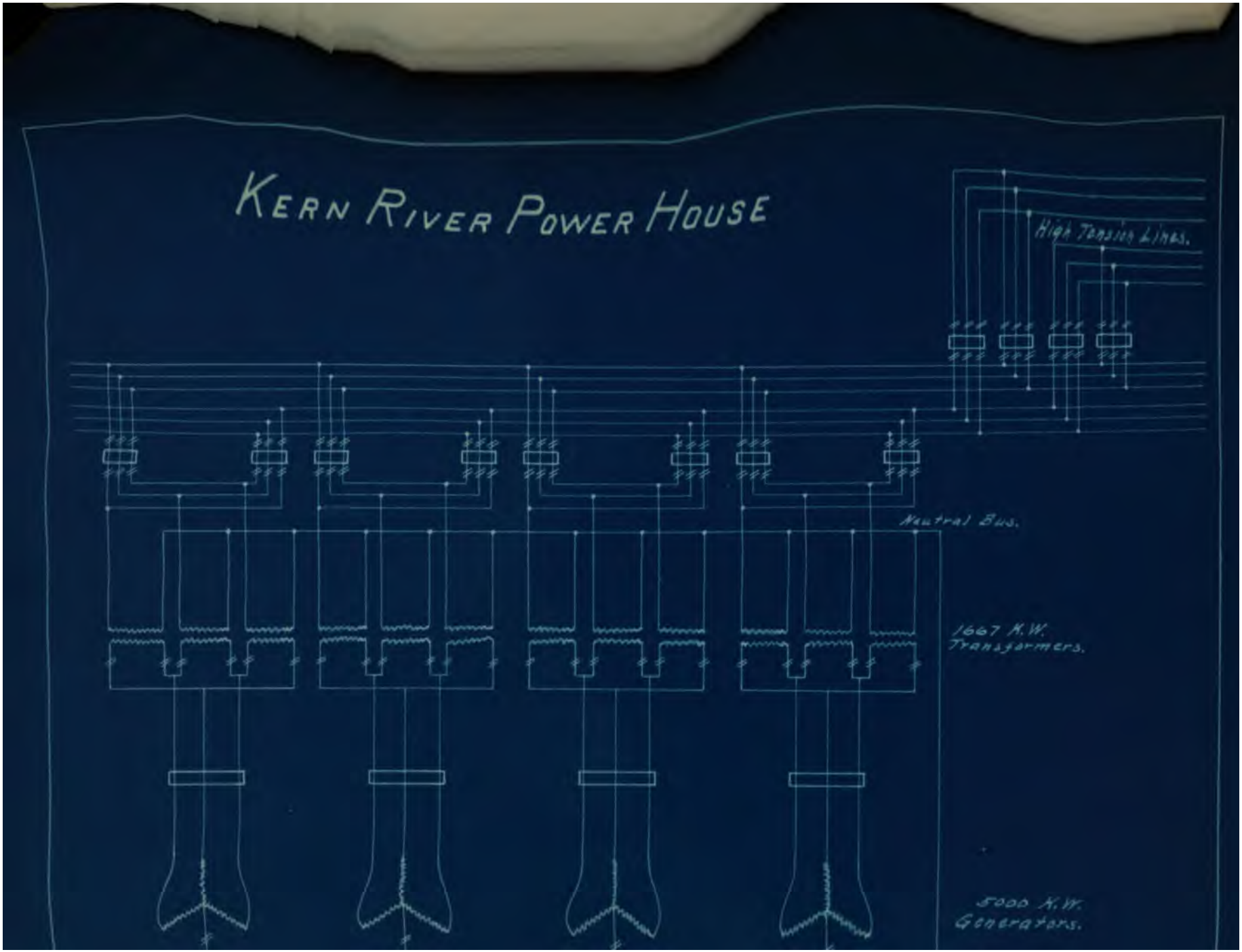
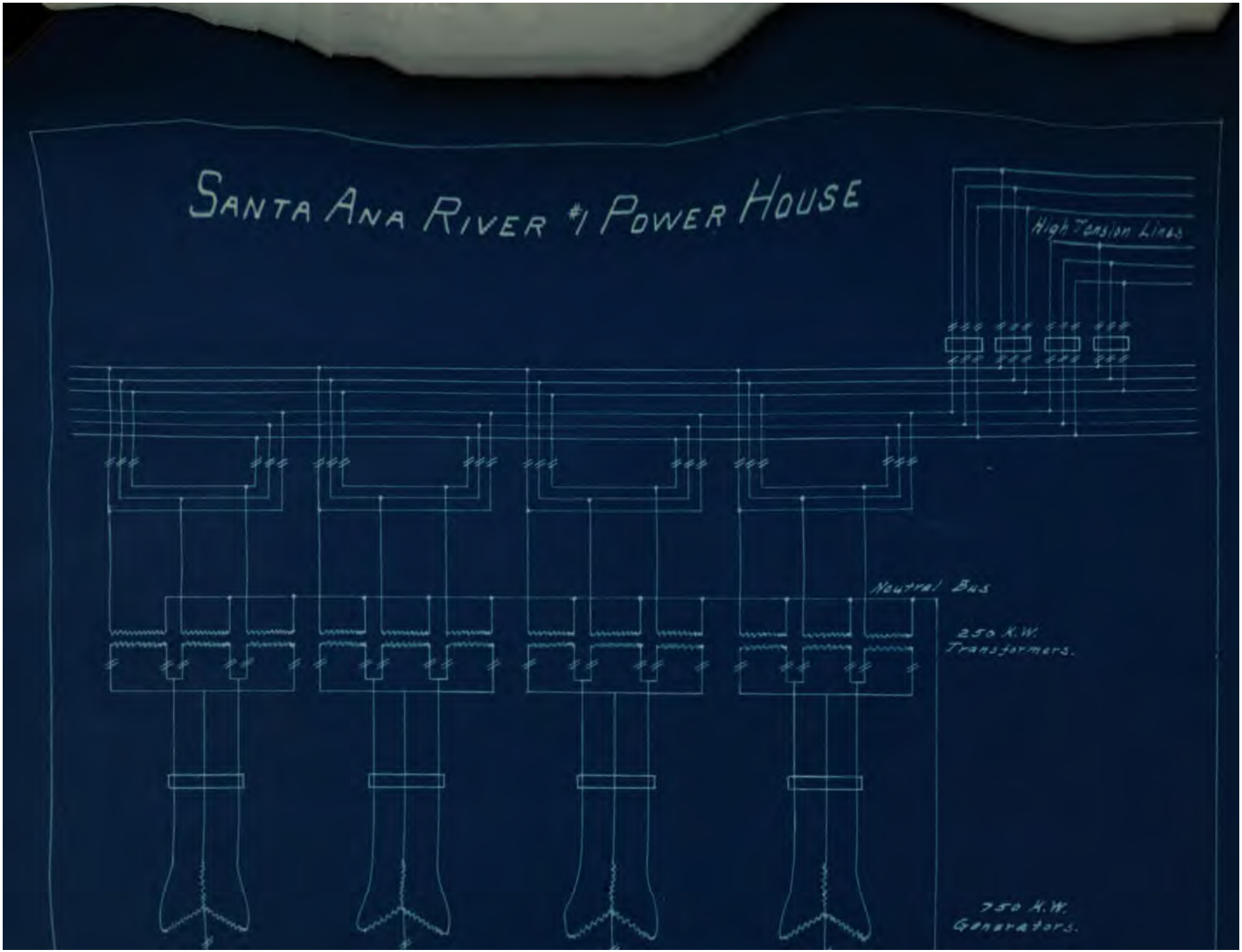
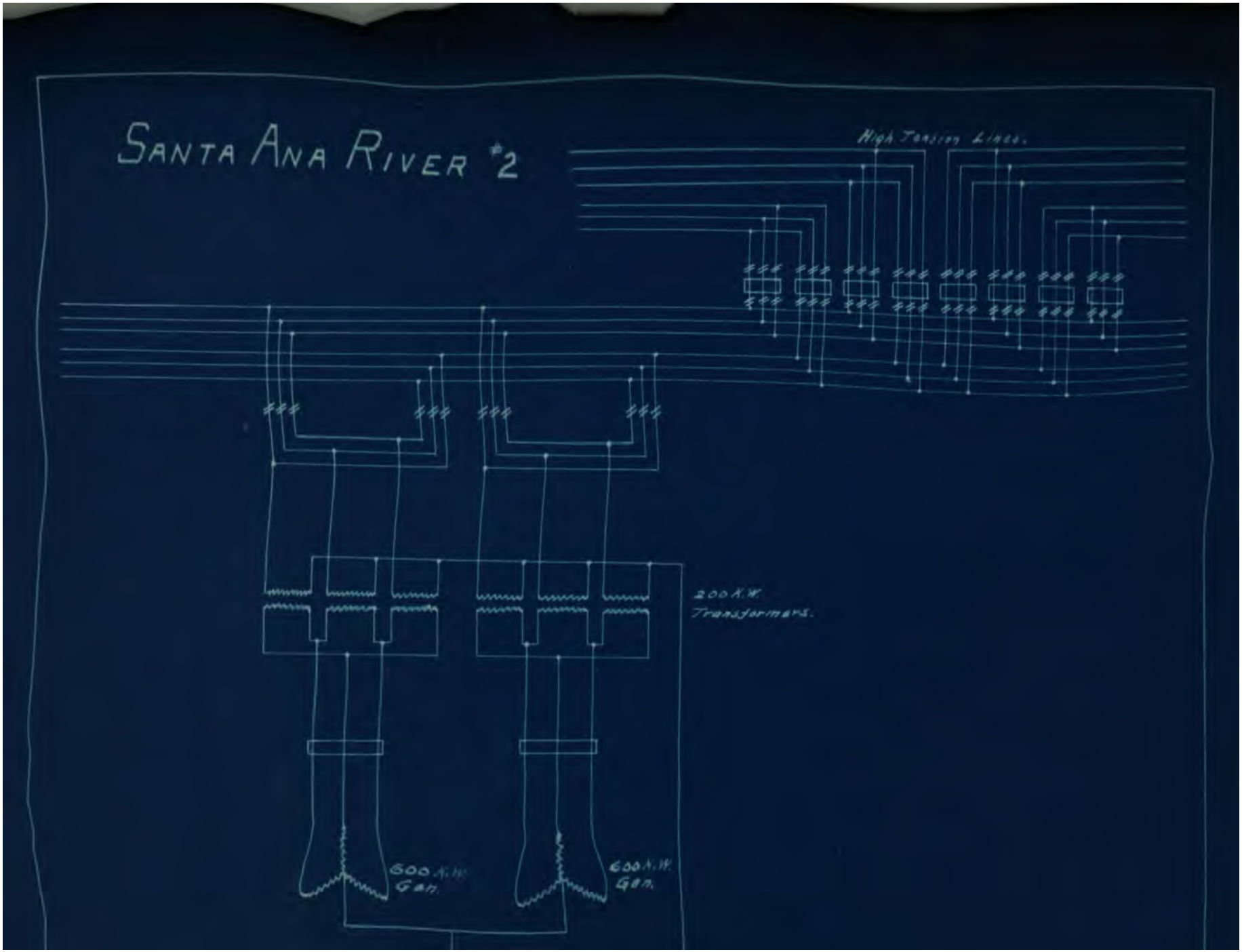
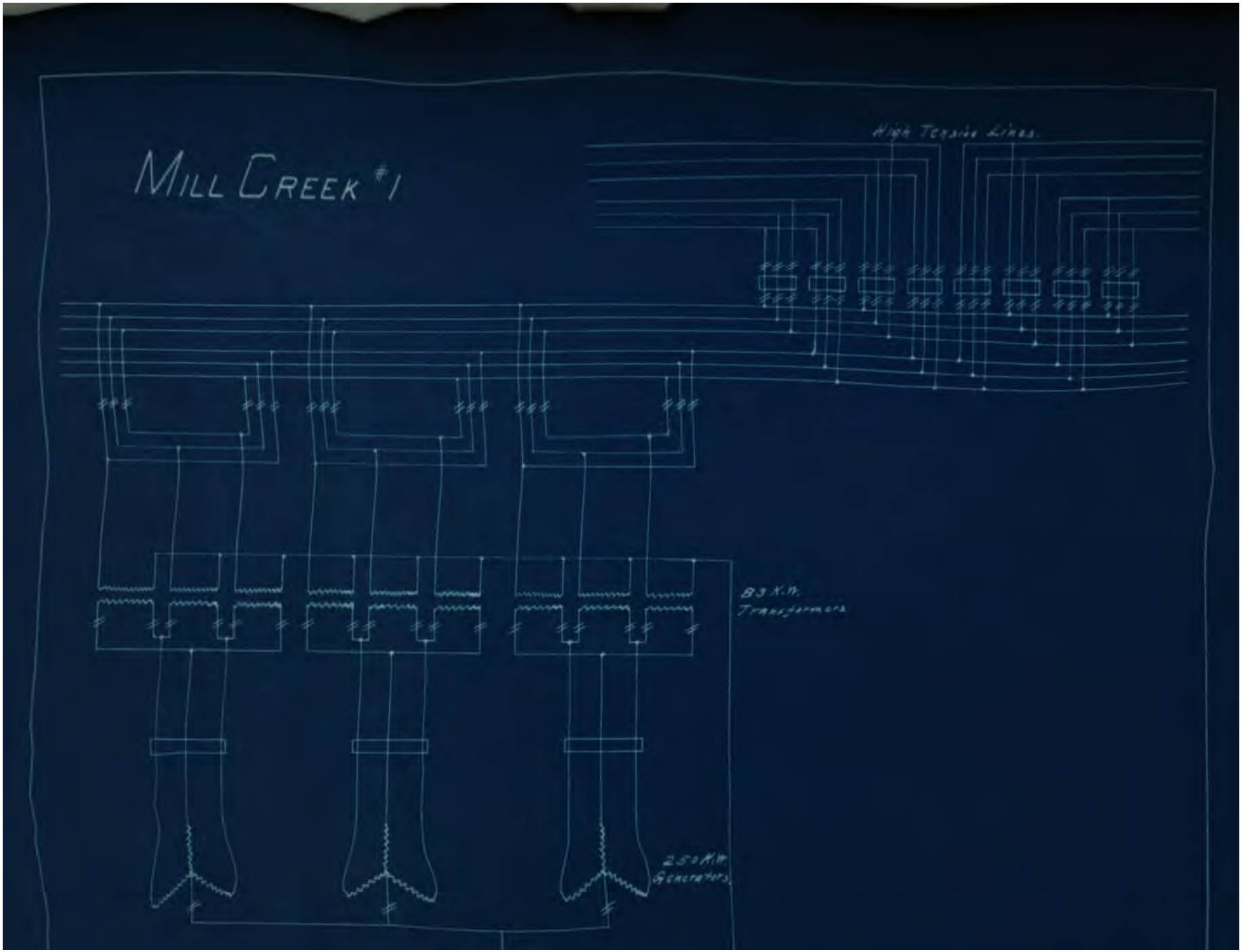

LOAD CHART
TOTAL MAXIMUM AND MINIMUM PEAK LOADS IN KILOWATTS
ENTIRE SYSTEM OF THE E. E. CO.
CHARACTERISTIC GENERATING CURVES
FOR
HIGHEST AND LOWEST PEAK DAYS
KILOWATTS
Average Total Load for Highest Peak Day
Total Load
Hydraulic Load
Steam Load
Day Load
Month of SEPTEMBER, 1902

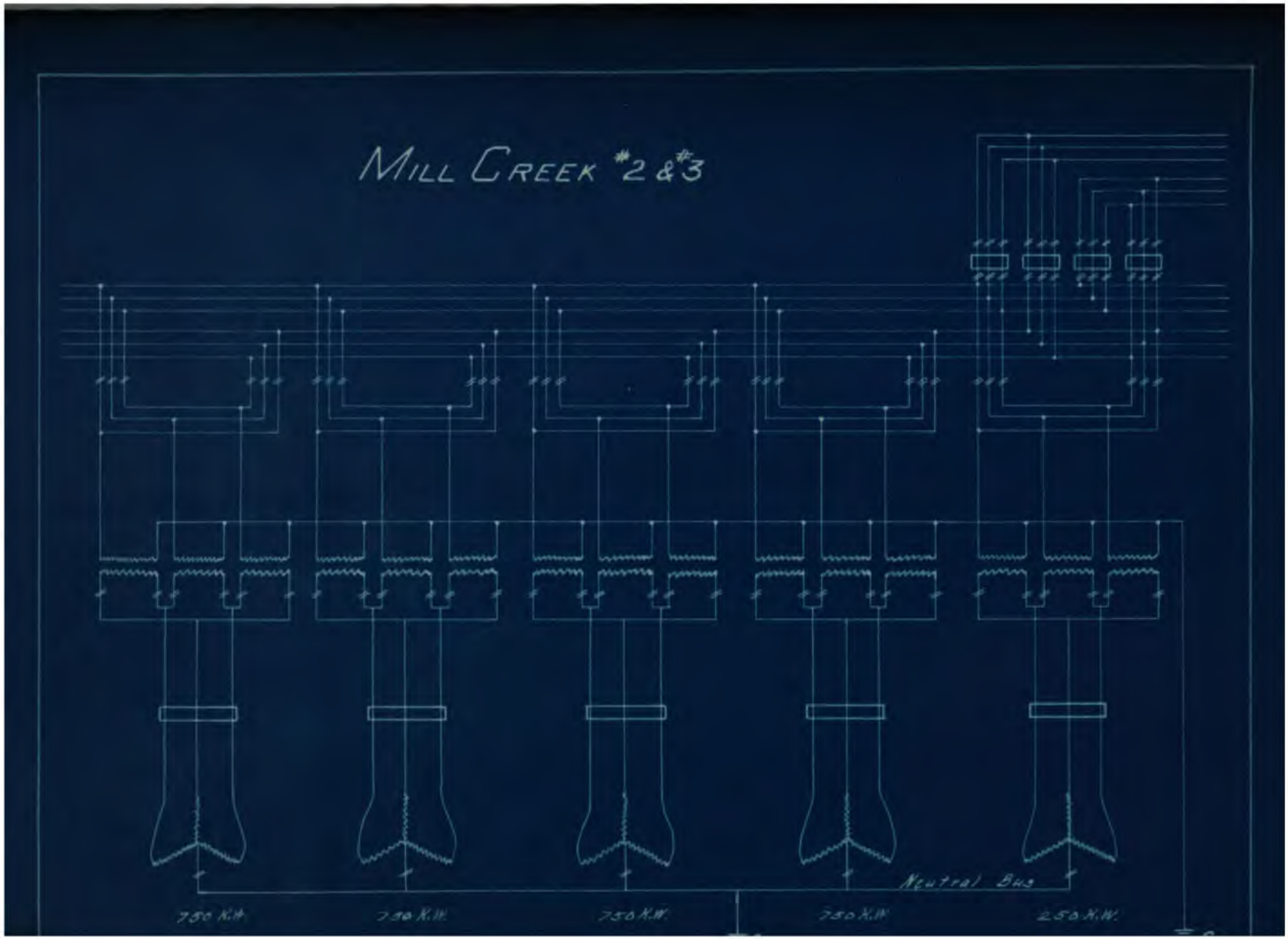
Plate IV.

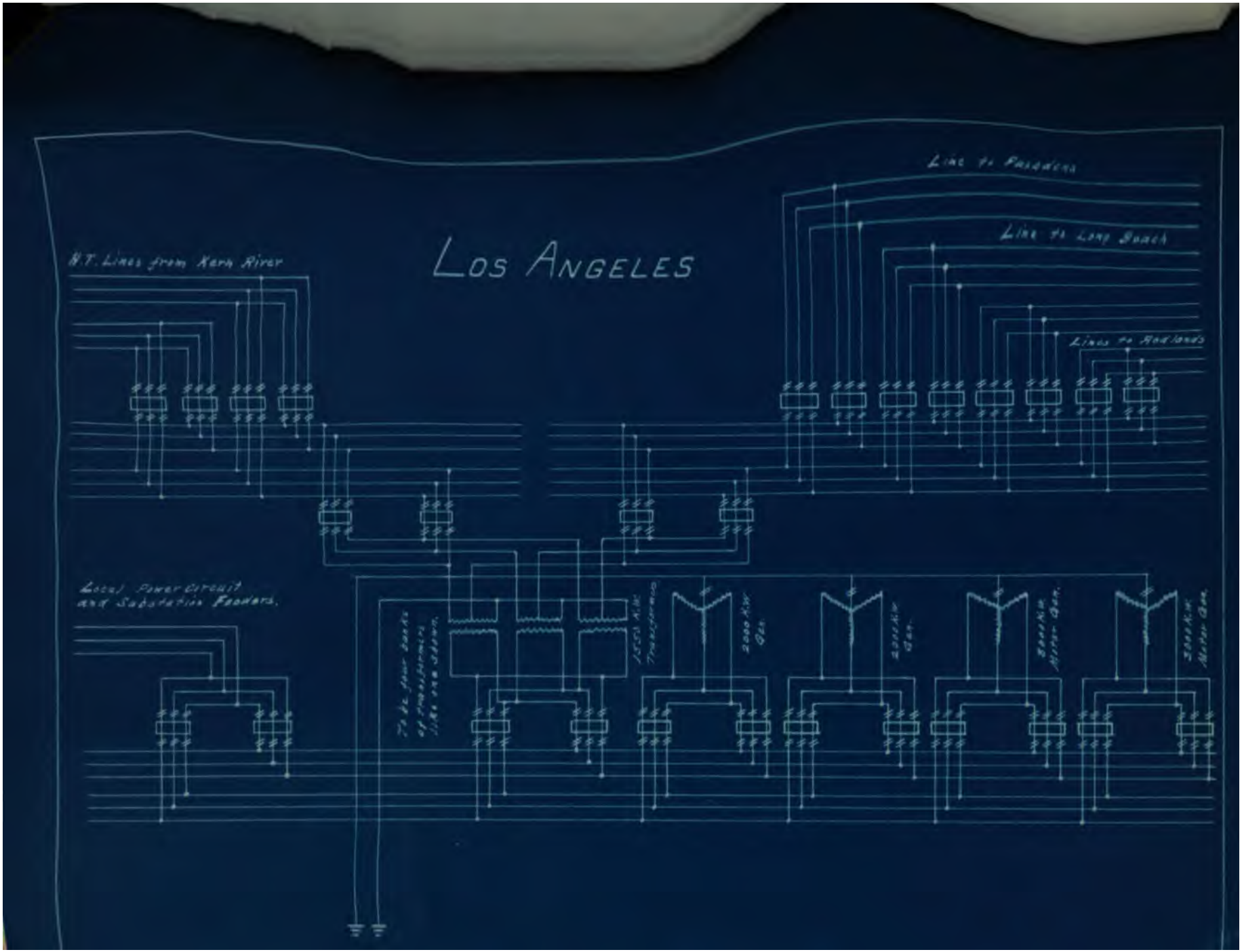

KERN RIVER POWER HOUSE
High Tension Lines.
Neutral Bus.
1667 K.W.
Transformers.
5000 K.W.
Generators.


SANTA ANA RIVER #1 POWER HOUSE
High Tension Lines
Neutral Bus
250 K.W. Transformers.
750 K.W. Generators.


SANTA ANA RIVER #2
High Tension Lines.
200 K.W.
Transformers.
600 K.W.
Gen.
600 K.W.
Gen.


MILL CREEK #1
High Tension Lines.
83 K.W.
Transformers
2500 K.W.
Generators.


MILL CREEK #2 & #3
NEUTRAL BUS
750 K.W.
750 K.W.
750 K.W.
750 K.W.
250 K.W.


Los Angeles
H.T. Lines from Kern River
Line to Pasadena
Line to Long Beach
Lines to Redlands
Local Power Circuit and Substation Feeders.
To 6 four banks of transformers 1,250 K.W. each.
1,250 K.W. Transformers
2000 K.W. Gen.
2000 K.W. Gen.
3000 K.W. Water Gen.
3000 K.W. Water Gen.

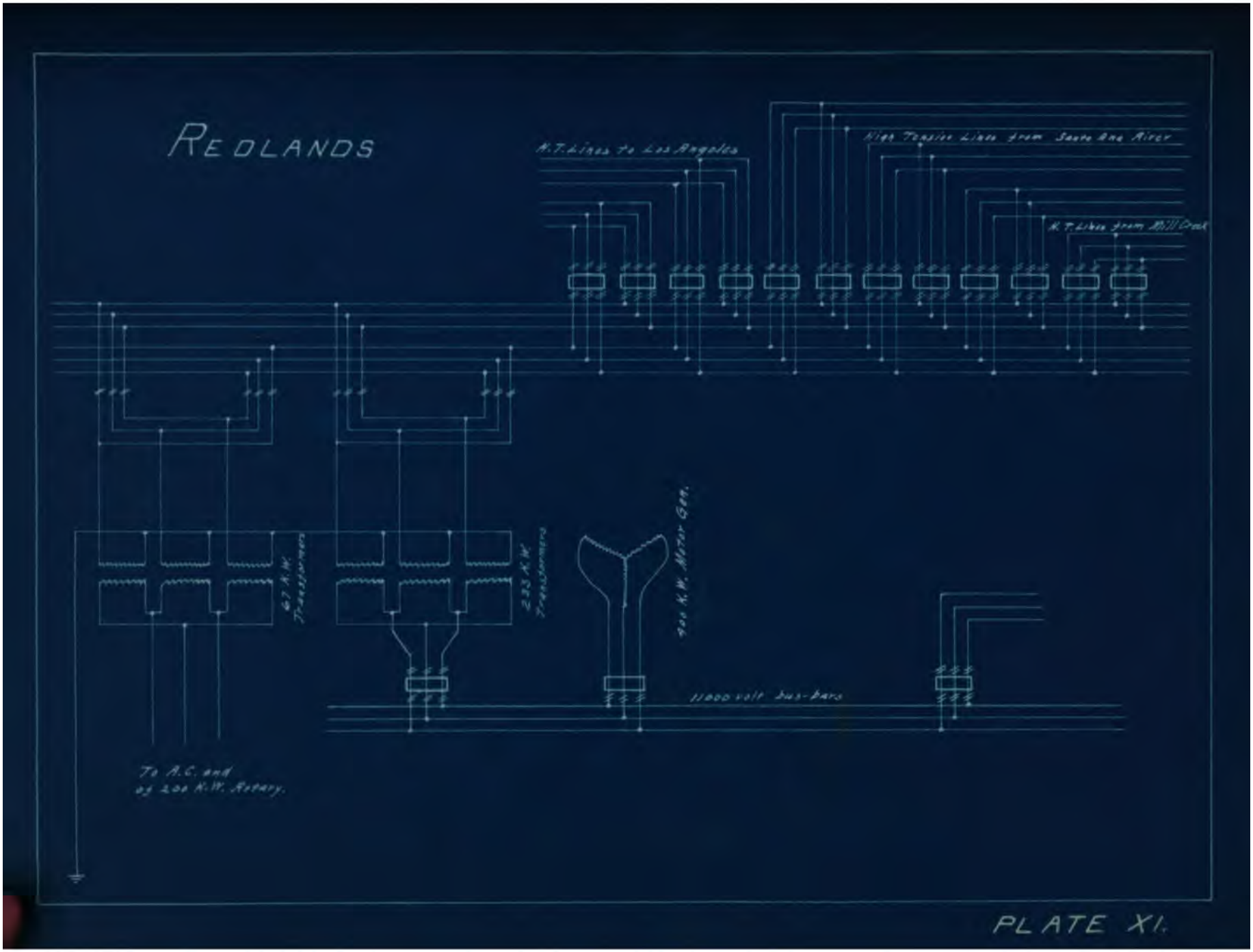
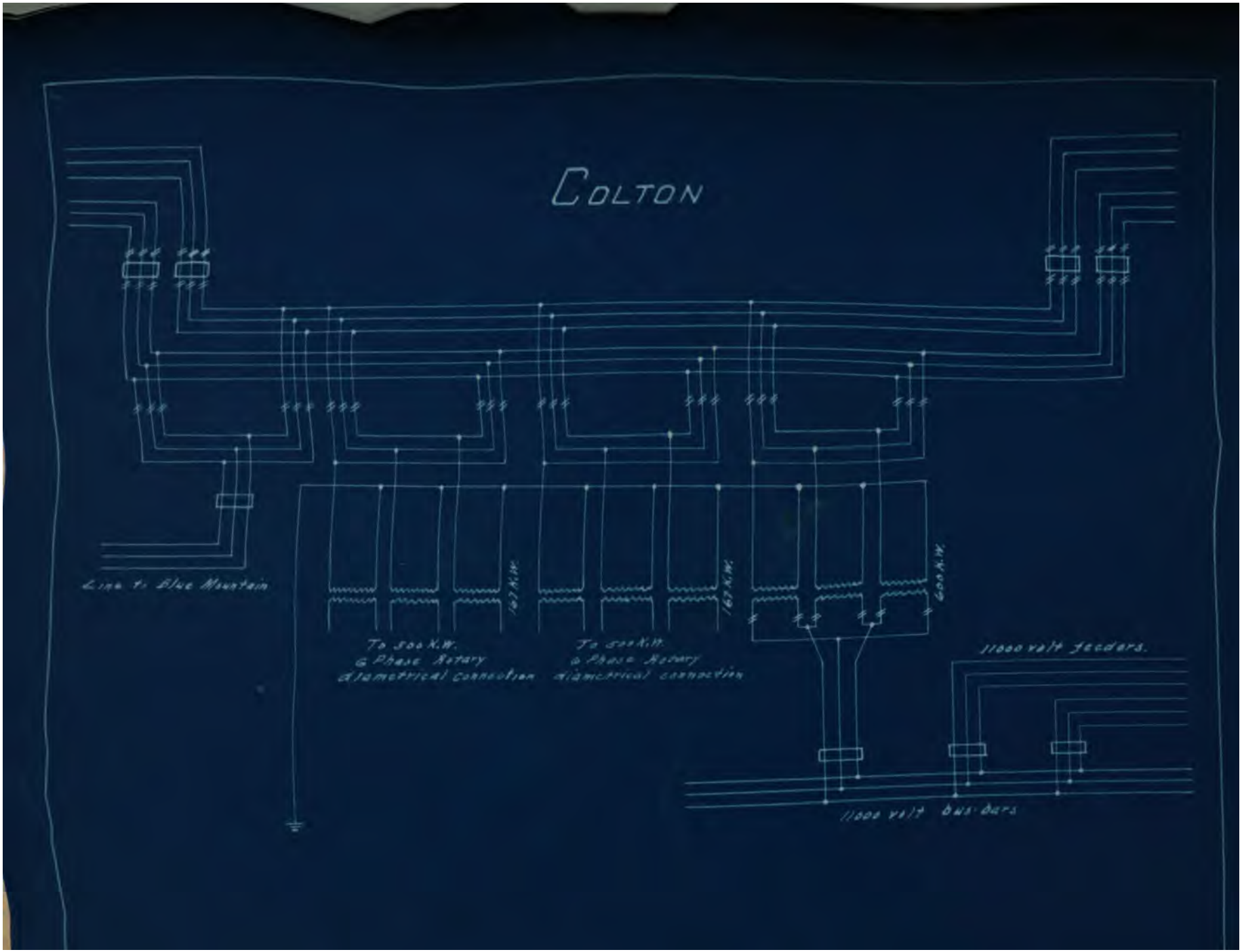
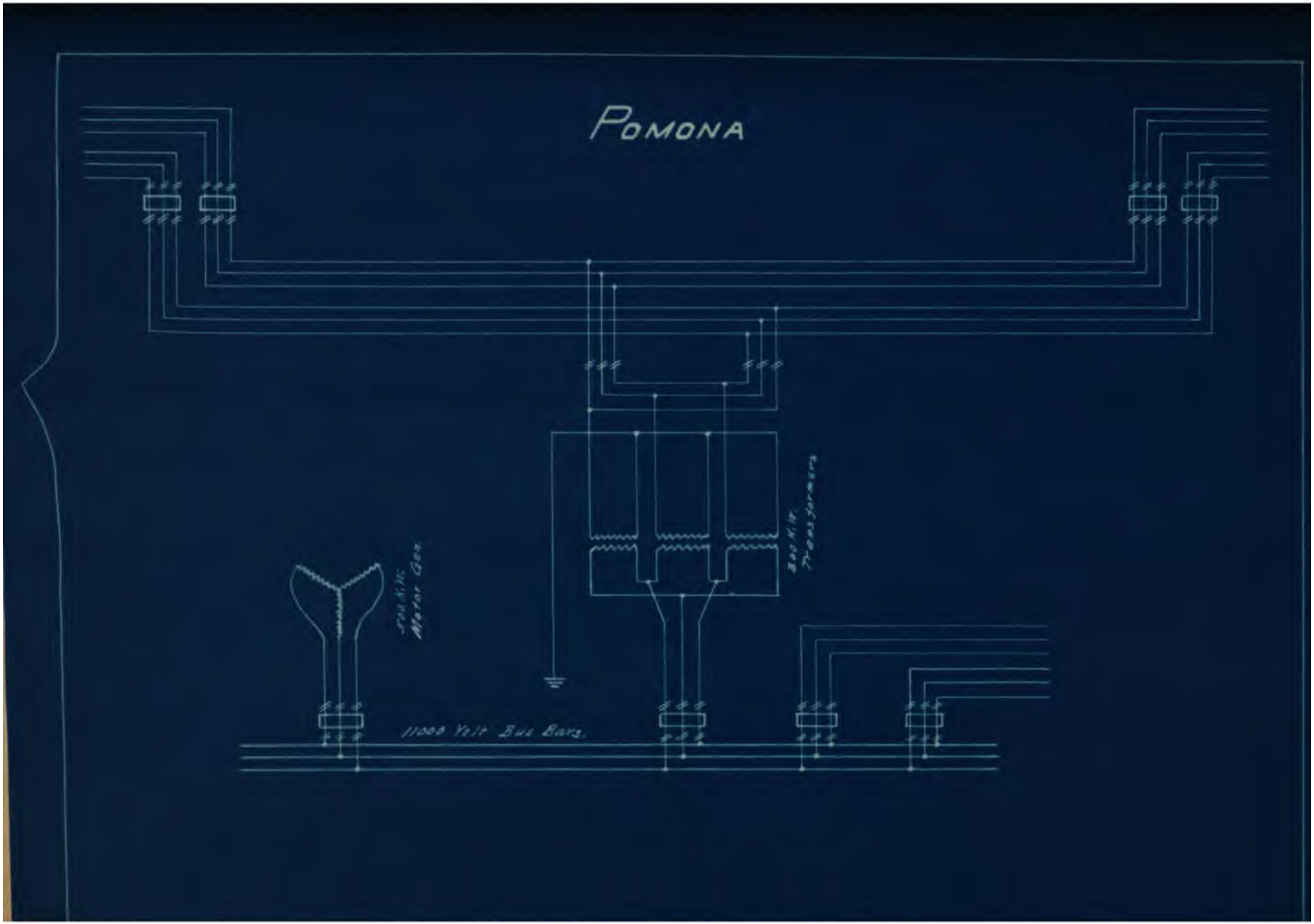

REDLANDS
H.T. Lines to Los Angeles
High Tension Lines from Santa Ana River
H.T. Lines from Mill Creek
67 K.W. Transformers
233 K.W. Transformers
400 K.W. Motor Gen.
11000 volt Bus-Bars
To A.C. and
25 200 K.W. Rotary.

PLATE XI.

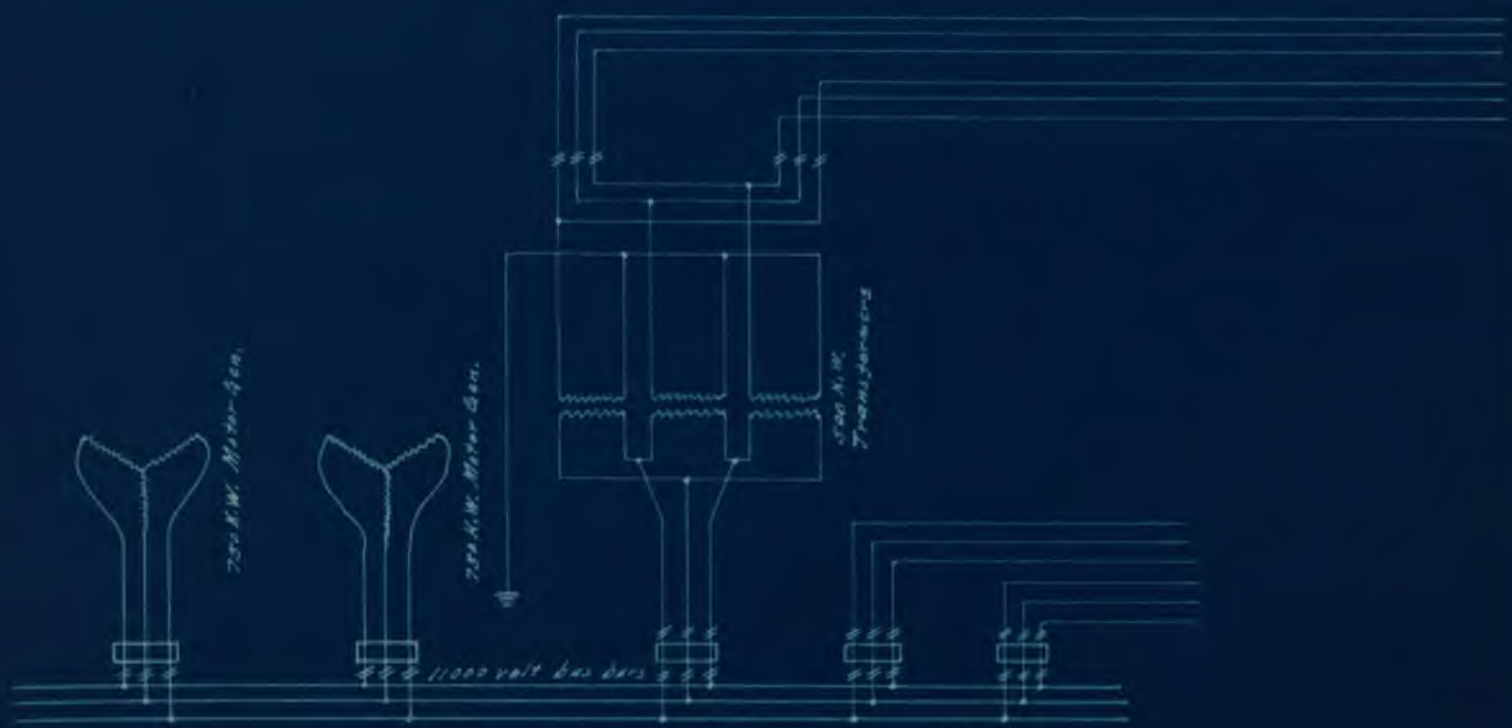

COLTON
Line to Blue Mountain
To 500 K.W. 6 Phase Rotary diametrical connection
To 500 K.W. 6 Phase Rotary diametrical connection
167 K.W.
167 K.W.
600 K.W.
11000 volt feeders.
11000 volt Bus-bars


POMONA
500 K.W. Motor Gen.
300 K.W. Transformers
11000 Volt Bus Bars.

PUENTE

PLATE XIV.

PASADENA


750 K.W. Motor-Gen.
750 K.W. Motor Gen.
1500 K.W. Transformers
11000 volt bus bars

89085930071

b89085930071a